Readers' Forum

Brief discussions of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Hypersonic Free Molecular Heating of Micron Size Particulate"

Nelson H. Kemp*
Physical Sciences Inc., Woburn, Mass.

In a recent Technical Note, Hall[1] considered free molecule heating of micron size particles at hypersonic speeds. In the course of the analysis he made use of the drag and heating expressions for spheres in free molecule flow taken from Ref. 2. The purpose of this Comment is to point out that the expression quoted by Hall from Ref. 2 for the recovery factor is erroneous. The error is only important for low values of the speed ratio (Mach number) so it probably does not affect Hall's results, which are for hypersonic flow. However, the error appears in so many standard works on free molecule flow that it seems worthwhile to point it out.

In fact, it seems that the formulas for drag and heating of spheres in free molecule flow have suffered more than the usual number of misprintings. Not only is the heating formula wrong in Ref. 2, it is also wrong in the related books, Refs. 3 and 4. A different incorrect version appears in Ref. 5. Furthermore, the drag formula is in error in Refs. 3 and 5, although Hall has used the correct version in Ref. 1.

On the other hand, some of the earlier papers in the field presented the correct formulas. As examples one can cite the papers of Sauer[6] and Oppenheim[7] for heating rate, and Stalder and Zurick[8] for drag.

From these works, or by following the derivation in Ref. 3 for example, one can obtain the heating rate to a sphere of radius a and surface temperature T_w, in a stream of density ρ, temperature T, and speed U, consisting of a gas with gas constant R and specific heat ratio γ. The expression is

$$Q=4\pi a^2\rho UR\alpha(T_r-T_w)St'(\gamma+1)/(\gamma-1) \tag{1}$$

where α is the thermal accommodation coefficient, T_r the recovery temperature, and St' the modified Stanton number. The latter two quantities are expressed in terms of the speed ratio S, which is related to the Mach number by

$$S=(\gamma/2)^{1/2}M=U/(2RT)^{1/2} \tag{2}$$

It is the ratio of flow speed to the most probable random speed in the flow. St' and T_r are correctly expressed as

$$St'=[S\,\mathrm{ierfc}(S)+S^2+0.5\,\mathrm{erf}(S)]/8S^2 \tag{3}$$

$$T_r=T[1+S^2r'(\gamma-1)/(\gamma+1)] \tag{4}$$

where r' is the modified recovery factor

$$r'=\frac{(2S^2+1)[1+S^{-1}\mathrm{ierfc}(S)]+(1-S^{-2}/2)\,\mathrm{erf}(S)}{S^2+S\,\mathrm{ierfc}(S)+0.5\,\mathrm{erf}(S)} \tag{5}$$

The error that appears in Ref. 2, Ref. 3 [Eq. (6-5)], Ref. 4 [Eq. (36)], and is repeated by Hall[1] in his Eq. (3), is in the last term in the denominator of r', Eq. (5), where they have a factor of $1/S^2$. This factor magnifies the importance of that term at low values of S, and causes $r'\to 0$ as $S\to 0$, instead of the correct limit $r'\to 8/3$. The error in Ref. 5 [Eq. (10. 5. 23)] is in St', where the factor S has been omitted from the first term in the numerator. The expressions given here are in agreement with the spherical results in Table 1 of Ref. 7, and also with the monatomic ($\gamma=5/3$) and diatomic ($\gamma=7/5$) results of Ref. 6. Hall and others frequently use more conventional expressions for Stanton number and recovery factor, which do not have the primes on the symbols, and are related to the modified expressions defined above by

$$St=\alpha St'(\gamma+1)/\gamma, \quad r=r'\gamma/(\gamma+1) \tag{6}$$

The advantage of the modified quantities is that they depend only on the speed ratio S, not on γ or α.

For completeness, the correct expression for a sphere drag coefficient will also be presented. It is based on the cross-sectional area of the sphere, πa^2, and depends on the tangential and normal reflection coefficients of the surface, denoted by σ and σ' respectively.

$$C_D=\frac{(2-\sigma'+\sigma)}{2S^3}\left\{\frac{4S^4+4S^2-1}{2S}\,\mathrm{erf}(S)+\frac{2S^2+1}{\sqrt{\pi}}e^{-S^2}\right\} \tag{7}$$

This expression is the same as Hall gave in his Eq. (1). However, it has been written erroneously in Ref. 3 [Eq. (8-6)] and in Ref. 5 [Eq. (10. 5. 21).] The error is in the exponent of the last term, which is given in these places as $S^2/2$ instead of S^2. This error has been recently pointed out in this journal by Henderson.[9] The formula is correctly stated in Ref. 8, for example.

Finally, simple alternate forms of St' and r' will be pointed out that do away with the awkward iterated complementary error function ierfc in favor of the ordinary error function, which is well-tabulated and available as a library routine on many computers. The relation between them is

$$\mathrm{ierfc}(z)=e^{-z^2}/\sqrt{\pi}-z[1-\mathrm{erf}(z)]$$

By using this in Eq. (3), we can write

$$St'=[Se^{-S^2}/\sqrt{\pi}+(S^2+0.5)\,\mathrm{erf}(S)]/8S^2 \tag{8}$$

Similarly, the modified recovery factor r' of Eq. (5) becomes

$$r'=\frac{(2S^2+1)[S^2\mathrm{erf}(S)+Se^{-S^2}/\sqrt{\pi}]+(S^2-0.5)\,\mathrm{erf}(S)}{S^2[Se^{-S^2}/\sqrt{\pi}+(S^2+0.5)\,\mathrm{erf}(S)]}$$

$$=\frac{2S^2+1}{S^2}-\frac{\mathrm{erf}(S)/S^2}{Se^{-S^2}/\sqrt{\pi}+(S^2+0.5)\,\mathrm{erf}(S)}$$

Received Nov. 6, 1978.
Index categories: Supersonic and Hypersonic Flow; Radiatively Coupled Flows and Heat Transfer.
*Principal Scientist. Associate Fellow AIAA.

Then Eq. (8) shows that

$$r' = 2 + \frac{I}{S^2} - \frac{\operatorname{erf}(S)}{8S^4(St')} \tag{9}$$

Equations (8) and (9) seem particularly simple to use in Eqs. (4) and (1) to give the heat transfer rate to spheres in free molecule flow.

Two other references have recently come to my attention with errors in the recovery factor. One is V. P. Shidlovskiy, *Introduction to Dynamics of Rarefied Gases*, American Elsevier, New York, 1967. On page 41, Eq. (2.52), the factor in front of erfs in the numerator should be divided by 2. The second is S. A. Shaaf," Mechanics of Rarefied Gases," in *Handbuch Der Physik*, Vol. III, No. 2, Springer-Verlag, Berlin, 1963. On page 605, Eq. (7.6), the last factor in the denominator should be multiplied by S^2.

Acknowledgment

The authors' colleague G. E. Caledonia pointed out the error in r' in Ref. 4 and suggested writing this Comment.

References

[1]Hall, D. R., "Hypersonic Free-Molecular Heating of Micron Size Particulate," *AIAA Journal*, Vol. 16, Aug. 1978, pp. 857-859.

[2]Schaaf, S. A. and Talbot, L., "Handbook of Supersonic Aerodynamics," Navord Rept. 1488, Feb. 1959.

[3]Schaaf, S. A. and Chambre, P. L., "Flow of Rarefied Gases," *Fundamentals of Gas Dynamics*, edited by H. E. Emmons, Princeton University Press, Princeton, N.J., 1958, Sec. H.

[4]Schaaf, S. A., "Heat Transfer in Rarefied Gases," *Developments in Heat Transfer*, edited by W. M. Rohsenow, The M.I.T. Press, Cambridge, Mass., 1964, Chap. 7.

[5]Hayes, W. D. and Probstein, R. F., *Hypersonic Flow Theory*, Academic Press, New York, N.Y., 1959, Chap. 10.

[6]Sauer, F. M., "Convective Heat Transfer from Spheres in a Free-Molecule Flow," *Journal of the Aeronautical Sciences*, Vol. 18, May 1951, pp. 353-354.

[7]Oppenheim, A. K., "Generalized Theory of Convective Heat Transfer in Free-Molecule Flow," *Journal of the Aeronautical Sciences*, Vol. 20, Jan. 1953, pp. 49-58.

[8]Stalder, J. and Zurick, V., "Theoretical Characteristics of Bodies in a Free Molecule Flow Field," NASA TN 2423, 1951.

[9]Henderson, C. B., "Reply by Author to M. J. Walsh," *AIAA Journal*, Vol. 15, June 1977, pp. 894-895.

Reply by Author to N.H. Kemp

Dennis R. Hall*
Massachusetts Institute of Technology,
Lexington, Mass.

THE author is happy that his Technical Note[1] has inspired Dr. Kemp to correct many erroneous printings of the free-molecular heating and drag equations which were published and republished by many people over the last two decades. The error in the recovery factor does, as he implied, have negligible effect on the predicted temperatures of Ref. 1. In particular, for a speed ratio of 13, the recovery used there is ≅0.2% higher than it would be using the correct recovery factor equation of Dr. Kemp. As s becomes bigger the error becomes smaller. On the other hand, if the tests were conducted near speed ratios of 2 to 4 the author would have made obvious temperature mispredictions.

Received Dec. 8, 1978.

Index categories: Supersonic and Hypersonic Flow; Radiatively Coupled Flows and Heat Transfer.

*Engineer, Aerodynamics, Aerospace Division, Lincoln Laboratory.

References

[1]Hall, D. R., "Hypersonic Free-Molecular Heating of Micron Size Particulate," *AIAA Journal*, Vol. 16, Aug. 1978, pp. 857-859.

Rebuttal to Reply by Author to "Comment on 'Flutter of Flat Finite Element Panels in a Supersonic Potential Flow' "

William P. Rodden*
La Cañada Flintridge, Calif.

DR. Yang's criticisms in his Reply[1] to my Comment[2] on his paper[3] were primarily limited to the structural aspects of the problem, and he replied to points most of which were conceded in the Comment. The matter of aerodynamic influence coefficients (AICs), however, deserves a few additional remarks. The AICs, as Revell and I have defined them,[4] constitute an aerodynamic finite element counterpart to the structural displacement method since they both relate forces to displacements. Nelson and Cunningham's work[5] certainly was published before my thesis, but they solved the stability problem by a routine application of the Galerkin method. By 1956, however, the validity of the Galerkin method had been called into question by a number of investigators because of an apparent paradox in analyzing the membrane panel (i.e., a panel with no bending stiffness) which is stable. Hence, the novelty of my thesis was found in a completely different approach, i.e., using both the structural and aerodynamic influence coefficients, which demonstrated the correctness of the Nelson and Cunningham Galerkin solution and contributed to a better understanding of the Galerkin method. Dr. Yang would not concede that my two-dimensional panel AICs preceded his (by 20 years) or that they were based on a more efficient computational algorithm. The point of referencing my thesis, or better yet, the *AGARD Manual on Aeroelasticity*[6] which discussed my AICs, is that this is where the first use of AICs appears in panel flutter analysis and the AICs included the *second order* frequency term.

Dr. Yang depreciates my treatment of the second order term when he replies: "When the simple trapezoidal rule can achieve excellent accuracy in approximating the aerodynamic pressure, the use of sophisticated higher-order numerical integration method is of no value, especially when the structural model is so crude." The remark on the crude structural model is, of course, irrelevant to any aerodynamic discussion, but Dr. Yang does not recognize the fact that he did not simply use a trapezoidal rule. He used the trapezoidal rule and then *an additional averaging* between grid points that increased his accuracy to be comparable to my "sophisticated" integration, but at some computational

Received July 1977; revision received Sept. 1978.

Index categories: Aeroelasticity and Hydroelasticity; Structural Dynamics; Supersonic and Hypersonic Flow.

*Consulting Engineer. Associate Fellow AIAA.